Dynamics Qualifying Exam – 2012

WORK ALL 5 PROBLEMS.

Problem 1

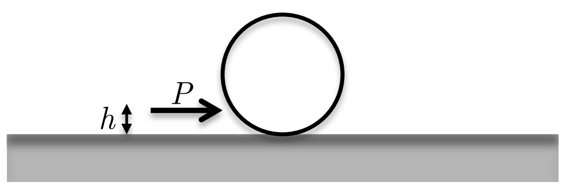
A uniform rope of length L and mass per unit length m lies on a frictionless horizontal table as shown. Initially, a length of rope L_0 overhangs the edge of the table. The acceleration due to gravity is g. The rope is released from rest at time $t = 0$.

(a) Determine the velocity of the end of the rope as a function of time.

(b) Determine the velocity of the rope when the back end just slides off of the table.

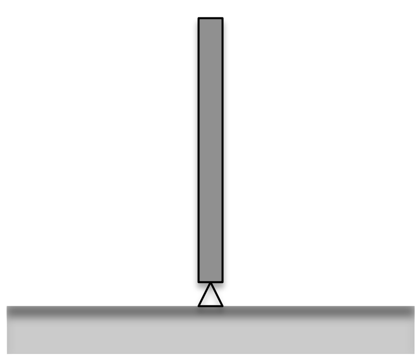
Problem 2

A homogeneous sphere of radius R and mass m sits at rest on a rough level surface. Suddenly, a large horizontal linear impulse of magnitude P is applied to the sphere at a height h above the level of the surface ($h<R$). The coefficient of friction between the sphere and the surface is μ. Find the distance x that the sphere travels while slipping on the surface.



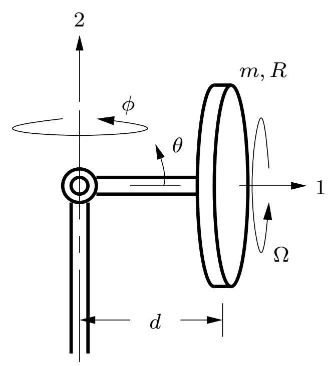
Problem 3

A uniform column of length L and mass m is tipped over. Its base is hinged to the ground. Determine the location where the maximum bending moment within the column occurs.



Problem 4

A disk of mass m and radius R is supported at the end of an arm of length d, and is driven to spin at a constant angular velocity Ω relative to the arm. The other end of the arm is supported by a vertical column. The vertical column can rotate freely (without friction) about its own vertical axis, through an angle ϕ. When the arm is rotated to a non-zero angle θ relative to the horizontal, the entire assembly is seen to rotate about the vertical column axis. Neglect the mass of everything but the disk, and derive an equation that governs ϕ as the angle θ changes.



Problem 5

Two rods, each of length $L/2$, are each fixed at one end, and they are joined with a coupler of mass m as shown. They are made of the same material so their mass density ρ and elastic modulus E are the same. However, their cross-sectional areas A_1 and A_2 are different. Let the axial vibration of the left rod be represented as $u_1(x,t)$ and that of the right rod as $u_2(x,t)$, where x ranges from 0 to L. *Derive* partial differential equations and boundary conditions governing u_1 and u_2. Idealize the coupler as being located at the single point $x = L/2$.